

TOPICS FOR INVESTIGATION IN MATHEMATICS

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Teaching mathematics and maintaining student interest at times entails bringing something new and different into the classroom. Creating effective and challenging situations for students requires a great deal of work and teacher ingenuity. The teacher must not only attempt to spark the individual's interest in mathematics, attempt to focus in on the individual's abilities, and foster within students the ability to investigate and inquire into topics on their own, an ability they must rely on after completion of their formal education.

One means of accomplishing these tasks is to present the student with a selection of short topics otherwise not covered in class and to encourage students to do individual investigations. A resource of ideas for such activities is the focus of this paper.

Listed below are nearly 600 topics for projects, short reports, investigations, units, inquiry or enrichment for mathematics from the upper elementary level to beyond high school. These topics are categorized under the headings:

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| A. Applications, | D. Investigations/Theories/Systems, | G. Social/Historical Topics, and |
| B. Classic Investigations, | E. Famous persons in Mathematics, | H. Theorems/Problems/Proofs. |
| C. Constructions/Devices/Models, | F. Recreations, | |

For completeness there are some duplications between categories. Undoubtedly this list is not exhaustive. Most persons can and will identify topics which are not included in this list. You are encouraged to record these in the blank spaces provided.

Photocopy and share this list with your colleagues.

A. Applications

1. The principle of aerodynamics.
2. Special types of annuities
3. Mathematics and architecture
4. The calculation of area
5. Basic artillery fire by mathematics
6. Conic sections in astronomy
7. Locating heavenly bodies by coordinates
8. Empirical equations in biology
9. Valuation of bonds and bond tables
10. Geometry of bubbles and liquid film
11. Mathematics of soap bubbles
12. Catenary curve and the suspension bridge
13. Geometry of the catenary
14. Distribution of weight and the center of gravity
15. Making circle graphs relative to such topics as how students use their time during the day
16. Codes and cyphers
17. Applied conic sections (illustrations and models)
18. Crystallography
19. Electronic computers
20. Prismatic crystals
21. Mathematics of depreciation/amortization
22. The Law of Divisibility
23. The Law of Disorder
24. Equilibrium
25. Cost of food for the month of _____
26. Illustrations of how formulas are used
27. The theory of flight and geometry
28. How to be a successful gambler
29. Everyday geometry
30. Uses of geometry in _____
31. Geometry in nature
32. Geographical representation of data
33. Gravitational constant(s)
34. The Law of Growth
35. Organic growth
36. The application of the gyroscope to an engineers transit
37. The mathematics of the gyroscope
38. The use of the gyroscope to navigate continuously and independently of external means
39. Collect data and graph the distribution of height of students in the school
40. Flying Boat (ship, canoe, sailboat) hydrostatics

41. The use of the level
42. The use of curves to represent life expectancies
43. Actuarial mathematics of life insurance
44. Sensations of light and color and mathematics
45. Collection of data and graphs of lung capacity
46. Man, mathematics and machines
47. Applied mathematics in the machine shop
48. Measuring distances on the earth's surface
49. The straight line (or great circle) in measurement
50. Mapping methods
51. Map projections
52. The use of trigonometry in navigation
53. Geometry in navigation
54. Problems in navigation with constructed models
55. Spherical trigonometry and celestial navigation
56. Pictures of persons in different occupations and the mathematics they use
57. How mathematics is used in a specific occupation
58. Mathematics of optics
59. Single and double strung pendulums
60. Obstructed pendulums
61. Mechanics of the pendulum
62. Mathematics and perspective drawing
63. Geometry of perspective drawing
64. Mathematics of photography
65. Physical applications of integrals
66. The mathematical nature of physical laws
67. Analysis of mathematical terms as related to classical physics
68. Mathematics and the pilot
69. Mathematics of population growth
70. The logarithmic curve of population growth
71. Profit and loss
72. Geometry in the construction of the pyramids (or _____)
73. Quantum theory and its development
74. Illustrating and/or applying the concept of ratio
75. Reflection and refraction of light
76. Relativity
77. Rockets - satellites - orbits
78. Roulette, craps, poker, slot machines, etc.
79. Scale drawing and mathematics

80. The normal distribution and sea shells
81. The sextant
82. Mathematics of shell forms
83. Mathematics of interstellar space travel
84. The carpenter's square
85. Mathematics of surveying
86. Mathematics of bridges
87. Mathematics and telescopes
88. Time curves
89. The use of the transit
90. Applications of the Laws of Triangles
91. Variables in graphs, farming, auto mechanics, other occupations
92. Distribution of weight
93. Mathematics and weather forecasting
94. Mathematics/Geometry and weaving
95. Mathematics of the coopers trade (barrel making)
96. _____

B. Classic Investigations

1. The fly and spider problem
2. The Gergonne Point
3. The Golden Section
4. Graphical development of the sieve of Eratosthenes
5. Moebius Strip
6. Napier's Bones
7. The Nine Point Circle
8. Peaucellier's Cell
9. The Problem of Apollonius
10. Pythagorean Triples
11. The rule of false position
12. The rule of three
13. The Simson Line
14. The three problems of antiquity: Squaring the circle, Trisecting an angle, Duplication of a cube
15. The many unsolved problems of number theory
16. _____

C. Constructions/Devices/Models

1. Making an abacus
2. The angle mirror
3. Making instruments for trisecting an angle
4. Pictures of angles to estimate and measure size
5. Models to demonstrate methods of obtaining area
6. Model to demonstrate the area of a circle

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| 7. Model to show area of a trapezoid | 58. Making a slide rule | 39. Fractals |
| 8. Model to show area of a triangle | 59. Slide rule for positive and negative numbers | 40. Sierpinsky's Traiangle |
| 9. Build an astrolobe | 60. The quadratic slide rule | 41. Mandelbrodt's theorem |
| 10. Soap bubble tank and use | 61. Model to show how "64 = 65" in the cut-out checkerboard problem. Try finding similar solutions | 42. Graphing of first and second degree equations by electronics |
| 11. An electromagnetic calculator | 62. Simple surveying instruments (models and how used) | 43. The graphical method of Fourier Series |
| 12. The Aztec calendar | 63. Models to illustrate theorems | 44. The three-dimensional graph as applied to the reading of complex roots from a two-dimensional graph |
| 13. Model to show how change in radius affects the circumference of a circle | 64. Surveying transit | 45. Graph of square roots and cube roots |
| 14. Clinometer | 65. Transparency overlays of math concepts | 46. Geodesics |
| 15. Construct and analog computer | 66. Models or diagrams to show the four centers of a triangle and methods of determining them | 47. Geometries other than Euclid |
| 16. A binary/computer/counter | 67. Geometric construction of triangles | 48. The geometry of great circles on a sphere |
| 17. Making models of other computation devices (Napier's Bones) | 68. Model to show relationship of volume of cylinder and cone | 49. Hyperbolic functions |
| 18. Conic sections models | 69. Snapshots or magazine pictures illustrating geometric figures | 50. Hypercomplex numbers |
| 19. Devices for drawing conic sections | 70. _____ | 51. The hyperbolic paraboloid |
| 20. Compass constructions | | 52. Infinity |
| 21. Construction of dart board for practice in reading coordinates | | 53. A critical analysis of infinity |
| 22. Models of six basic types of crystalline formation | | 54. Infinite and Fourier Series |
| 23. Cylindrical projections | | 55. Game theory |
| 24. Variable cone cylinder | | 56. Mathematics of games |
| 25. Geodesic dome construction | | 57. On general invariants |
| 26. "Etch-a-sketch" and linear equations in two variables | | 58. Groups |
| 27. Diagrams to illustrate meaning of descending and ascending order of exponents | | 59. Index numbers |
| 28. Problems and illustrations of a number raised to a power, such as grains of wheat on a checkerboard or folding a newspaper | | 60. Method of least squares |
| 29. Model to illustrate fly-and-spider problem and its solution | | 61. Linkages |
| 30. Model to show geometric interpretation of $(a + b)^n$ | | 62. Lissajou's Figures |
| 31. Model to show geometric interpretation of $(a - b)^n$ | | 63. Lobachevskian Geometry |
| 32. Model to show geometric interpretation of $(a - b)^n$ | | 64. The determination of the locus of a point on a circle rolling on another circle which itself rolls on a straight line |
| 33. $(a+b)$ to the fourth power model | | 65. Logarithms and the slide rule |
| 34. Golden Section model | | 66. Various systems of logarithms |
| 35. A circle-graph | | 67. Logarithms of negative and complex numbers |
| 36. Space graph | | 68. The use and advantage of the LaRoix and Ragot graphic tables of logs |
| 37. 3-dimensional graphing | | 69. Logic |
| 38. Drawing visual 3-dimensional objects on 2-dimensional surface using mirror | | 70. Many valued logic |
| 39. Diagrams or models to illustrate the concept of limit | | 71. Symbolic logic for the layman |
| 40. Linkages | | 72. The nature of symbolic logic |
| 41. Locus, illustrations and models | | 73. Mascheroni's Constructions |
| 42. Indirect measurement (problem and models) | | 74. Matrices |
| 43. Models of various units of measure | | 75. The mathematics of Mendel's Theory |
| 44. $f(x) = a + b \sin(cx + d)$ model | | 76. Minimal surfaces |
| 45. Drawings or collections of optical illusions | | 77. Monte Carlo Method of number approximation |
| 46. A device for solving quadratic equations | | 78. Development of the Normal Law of deviations |
| 47. Quipu (South American native counting device) | | 79. Number systems and number symbols |
| 48. Making a pantograph and showing how it works | | 80. Operations with numbers in base twelve (eight, etc.) |
| 49. Plane tables | | 81. Patterns of numbers |
| 50. Building a planimeter | | 82. Perfect numbers |
| 51. Single cut polygons and construction of polygons | | 83. The number Pi, Phi, and/or "e" |
| 52. Typing polygons from strips of paper | | 84. Polygon family |
| 53. Model or chart to illustrate kinds of polygons (the Geom-e-tree) | | 85. An investigation of the properties of the diagnosis of polygons |
| 54. Polyhedra from straws | | 86. The regular seventeen-sided polygon |
| 55. Probability board | | 87. Regular polygons |
| 56. Inclines protractor | | 88. Polyhedra (cone in various methods) |
| 57. Model to show Pythagorean relationship | | 89. The five regular polyhedrons |
| | | 90. Prime numbers |
| | | 91. Projective geometry |
| | | 92. Probability |
| | | 93. Graph from matching coins compared with $(a + b)^n$ |
| | | 94. Charts to show how a proof is based on definitions, axioms, and postulates |

D. Investigations/Theories/Systems

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| 1. Mathematics and alchemy | 65. Logarithms and the slide rule |
| 2. Solid analytic geometry | 66. Various systems of logarithms |
| 3. Theory of arithmetical operations | 67. Logarithms of negative and complex numbers |
| 4. Axioms for the Constitution | 68. The use and advantage of the LaRoix and Ragot graphic tables of logs |
| 5. Original illustrations for meanings of axioms | 69. Logic |
| 6. Barycentric Calculus | 70. Many valued logic |
| 7. Boolean Algebra and its applications | 71. Symbolic logic for the layman |
| 8. Theory of Braids | 72. The nature of symbolic logic |
| 9. Casting out nines | 73. Mascheroni's Constructions |
| 10. Circular definitions - show how some dictionary definitions eventually lead to original word | 74. Matrices |
| 11. Approximate constructions | 75. The mathematics of Mendel's Theory |
| 12. A coordinate system based on a fixed circle | 76. Minimal surfaces |
| 13. Circular coordinates and the cardioid | 77. Monte Carlo Method of number approximation |
| 14. Geometric curves | 78. Development of the Normal Law of deviations |
| 15. Research in plane curve theory | 79. Number systems and number symbols |
| 16. Study of mathematical curves | 80. Operations with numbers in base twelve (eight, etc.) |
| 17. Three dimensional curves | 81. Patterns of numbers |
| 18. Investigating the cycloid | 82. Perfect numbers |
| 19. Negative-denary system | 83. The number Pi, Phi, and/or "e" |
| 20. Deduction in geometry | 84. Polygon family |
| 21. Mathematical designs by photographic methods | 85. An investigation of the properties of the diagnosis of polygons |
| 22. Geometric Dissections-Tangrams | 86. The regular seventeen-sided polygon |
| 23. Divisibility of numbers | 87. Regular polygons |
| 24. The Fourth Dimension | 88. Polyhedra (cone in various methods) |
| 25. Theories behind a mathematical fourth dimension | 89. The five regular polyhedrons |
| 26. Fourth dimension and concept of dimensionality | 90. Prime numbers |
| 27. The duodecimal system | 91. Projective geometry |
| 28. The construction of the complex roots of equations | 92. Probability |
| 29. Digraphs and their use | 93. Graph from matching coins compared with $(a + b)^n$ |
| 30. Diophantine equations | 94. Charts to show how a proof is based on definitions, axioms, and postulates |
| 31. Discreet mathematics | |
| 32. Two and three dimensional equations and graphs | |
| 33. Fallacies of mathematics | |
| 34. Fibonacci Numbers | |
| 35. Fields | |
| 36. Finger reckoning | |
| 37. Finite differences | |
| 38. A finite number system | |

95. Geometric construction of quadratic equation
96. Normalizing the quadratic equation
97. Quadratic surfaces
98. Rates of change
99. Recursion
100. Nongeometric illustrations of indirect reasoning (including some puzzles)
101. On the solution of the Ricatti equations by the LaPlace transformation
102. Riemannian Geometry
103. Sequences and the number "e"
104. Summation of sequences
105. Mathematical series and vanishing triangles
106. Theory of sets
107. Spirals
108. Fundamentals of statistics and statistical methods
109. Geometric Stereograms
110. Strategy
111. Interesting properties of mathematical surfaces
112. Ruled surfaces
113. Symmetry (meaning, types, illustrations, models)
114. Topology
115. The geometry of a tore
116. Tractenberg method
117. Geometric transformations
118. Spherical Triangles
119. Approximate computations with trigonometric functions and logarithms
120. Trigometric functions of imaginary and complex angles
121. Vectors
122. Studies in vector analysis
123. Negative base number systems
124. _____

E. Famous Persons in Mathematics

1. Abel
2. Adelard of Bath
3. Al-Biruni
4. Al-Fazari
5. Archimedes
6. Aristotle
7. Emil Artin
8. Aryabhata
9. Daniel Bernoulli
10. Jean Bernoulli
11. Bhaskara
12. George David Birkhoff
13. William Blaschke
14. Anicus Manlius Severinus Boethes
15. Immanuel ben Jacob Bonfils
16. Harald Bohr
17. George Boole
18. George Ferdinand Ludwig Phillip Cantor
19. Hieronimo Cardan
20. Elie Carten
21. Arthur Caylay
22. Chin Chiu-Shao
23. Copernicus
24. Augustin-Louis Cauchy
25. Cavalieri
26. Pafnutid Lvovitch Chebychev
27. Alexis-Claue Clairaut
28. Leonardo De Vinci

29. Julius Wilhelm Richard Dedekind
30. Abraham DeMoivre
31. Gerard Desargues
32. Rene Descartues
33. Albrecht Durer
34. Albert Einstein
35. Leonhard Euler
36. Euclid
37. Guilio Carlo deToschi de Fagnano
38. Piere-Simon De Fermat
39. Fibonacci
40. Galileo
41. Galois
42. Johann Friedrich Carl Gauss
43. Gelfond
44. James Gregory
45. William Rowan Hamilton
46. Jacques S. Hadamard
47. Godfrey Harold Hardy
48. Charles Hermite
49. David Hilbert
50. Christiaan Huygens
51. Carl Gustav Jacob Jacobi
52. Jevon
53. Mohammed Abu Bekr Ibn Al-Hasan Al Karkhi
54. John Kepler
55. Omar Khayyam
56. Al Khowarizmi
57. Saki Kowa (Takakatu)
58. Leopold Kronecker
59. Joseph Louis Lagrange
60. Johann Heinrich Lambert
61. Laplace
62. Emmanuel Lasker
63. Henri Lebesgue
64. Legendre
65. Gottfried Wilhelm Leibniz
66. Lejeune-Dirichlet
67. Marius Sophus Lie
68. Joseph Liouville
69. Nikoli Ivanovitch Lobashevsky
70. Colin MacLaurin
71. Benoit B. Mandelbrot
72. Hermann Minkowski
73. Gaspard Monge
74. John Napier
75. Nicholas of Cusa
76. John Von Neumann
77. Isaac Newton
78. Emy Noether
79. Oresme
80. Lucas Paciulus
81. Blaise Pascal
82. Pegolatti
83. Emile Picard
84. Jules Henri Poincare
85. Charles de La Vallee Poussin
86. Pythagoras
87. Ramanujan
88. Regiomontanus
89. Riemann
90. Bertrand Russell
91. Saccheri
92. Schroder
93. Waslaw Sierpinsky
94. Simon Stenin
95. James Joseph Sylvester
96. Pope Sylvester II
97. TeijiTakagi

98. Tartaglia (Nicolo Fontana)
99. BrookTaylor
100. Thales
101. Oswald Veblen
102. Franciscus Vieta
103. Vinogradov
104. Vito Volterra
105. John Wallis
106. KarlTheodor Wilhelm Weierstrase
107. Herman Weyl
108. John H. C. Whitehead
109. Norbert Wiener
110. William of Occam
111. Christopher Wren
112. Zeno
113. Zermelo
114. _____

F. Philosophy/Art/Music

1. Mathematics as an abstract creation
2. Mathematics in art
3. Mathematics as an art
4. Mathematics in Astrology
5. Mathematics and color
6. Color analysis
7. Cubism in art
8. Mathematics of design
9. Circle designs
10. Mathematical applications to philosophy exploration into the nature of complex numbers
11. Geometric designs for stained glass windows (with cellophane and black tape)
12. Geometric designs on paper plate
13. Original designs worked out in Cartesian coordinates
14. Geometric figures useful in decorations
15. The relation between the mathematical and the philosophical work of Descartes or Leibniz
16. The notion of dimensions
17. The Golden Section
18. The source of mathematical ideas
19. The notion of infinity in the work of the Latin poet, Lucretius
20. Influence of mathematics on Philosophy
21. Influence of Philosophy on mathematics
22. Influence of relation on mathematics
23. Mathematics as religion
24. Influence of science on mathematics
25. Influence of mathematics on science
26. Influence of mathematics on society
27. Influence of society on mathematics
28. How we play mathematics
29. Our sensations of light and color
30. An analysis of the tone quality of familiar musical instruments
31. Science and musical sounds
32. Sound and Music
33. Mathematics and Music
34. Musical scales
35. The physical basis of music
36. Twelve-tone music and mathematics
37. Geometric Christmas ornaments
38. The operation theory of meaning
39. Various views of the nature of mathematics
40. The place of mathematics in the formation of physical theories

42. Current beliefs regarding the extent of the physical universe
43. Relativity and Mathematics
44. The senses as reliable sources of knowledge
45. Dynamic symmetry
46. Dynamic symmetry and its practical applications
47. Tessellations
48. Philosophy of Zeno
49. _____

G. Recreations

1. Mathematical cartoons (collect from magazines or originals)
2. Mathematics and chess
3. Curve stitching
4. Flexagons
5. Paper folding/origami
6. Hexaflexagons
7. Mathematical oddities
8. Geometric puzzles
9. Puzzles involving algebra (such as the tower problem involving transfer of discs)
10. Magic squares/Latin squares
11. Magic squares developed with positive and negative numbers and polynomials
12. _____

H. Social/Historical Topics

1. Chinese abacus
2. Historical methods of adding, subtracting, and multiplying integers
3. History of algebra
4. History of Arabic Numbers
5. History of arithmetic
6. Ancient astronomy
7. Chaucer and the astrolabe
8. The Calendar
9. History of Chinese mathematics
10. The Cissoïd of Diocles and Witch Agnes
11. Ancient computing devices
12. A history of conic sections and a study of the ellipse
13. The Constitution of the United States as a set of postulates in politics
14. Coordinate geometry before Descartes
15. The relations between mathematical thought and the culture of a civilization
16. The invention and history of decimals
17. The resemblance of the American Declaration of Independence to a mathematical system
18. A historical study of methods of dividing integers
19. The use of mathematics, other than statistics, in economics, government or sociology
20. Mathematics of the Hindus and Arabs
21. History of Hindu Arabic system
22. History of geometry
23. History of Greek geometry
24. The groma
25. The hypsometer
26. Kepler-Poinsot polyhedra
27. The Egyptian level
28. History of logarithms
29. The Metric System
30. Napier's Bones

31. Mathematics in the news: collect clippings about math
32. Nomographs
33. Numbers on postage stamps
34. Names of numbers in different languages
35. Symbols of numbers
36. The history and practical applications of numerical systems
37. History of "Pascal's" triangle
38. Past and future according to the theory of relativity
39. Mathematics of the Phoenicians, Babylonians, and Egyptians
40. The influence of mathematics on political thought
41. The Pythagoreans
42. Egyptians used the theorem of Pythagoras
43. The quincunx
44. Quipu (South American native counting device)
45. Reasoning fallacies in advertising (showing implied conclusions)
46. Renaissance mathematics
47. The effects of modern science on religious thought
48. The slide rule: its history and operation
49. The influence of mathematics on the social sciences
50. The center square
51. Steinmetz and Electricity
52. The telemeter
53. The tesseract
54. History of the arithmetic text back to 1514
55. History of Trigonometry
56. _____

I. Theorems/Problems/Proofs

1. Brocard points
2. Cavalieri's Theorem
3. Graphical representation of Complex roots of quadratic and cubic equations
4. Desargues' Theorem
5. Proof of $E = mc^2$
6. Write original equations and solutions
7. Solving equations of the third degree and higher
8. My work on the equation: $ax + b = y$
9. Euler numbers
10. The Euler line
11. Extension of Euler's formula to N-dimensions
12. Fermat's Last Theorem
13. The Four-Color problem
14. Gergonne's Problem
15. A locus problem
16. Maxima and minima
17. Geometric and harmonic means
18. Demonstration of mixture problems
19. Multinomial Theorem
20. Extension of Pappus' Theorem
21. Attempts to prove the parallel axiom
22. Pascal's Theorem
23. Attempts to prove the parallel axiom
24. Theory of perspectives
25. Pi
26. Methods of estimating Pi
27. Circle to demonstrate value of Pi
28. Probability determination of Pi
29. Determining value of Pi by other methods

30. Pythagorean Theorem/Triples
31. Pythagorean Theorem (different proofs)
32. The use of the theorem of residues in the evaluation of certain definite integrals
33. Theorems on the roots of equations
34. A formula for calculating the resultant of two or more sine waves of the same frequency
35. Proof of the theorems of plane geometry by means of coordinates
36. Write original word problems and solutions
37. _____